GeoGebra as a Technological Tool in the Process of Teaching and Learning Geometry

Abdón Pari Condori[1](✉), Derling Jose Mendoza Velazco[2](✉), and Roxana Auccahuallpa Fernández[2](✉)

[1] Universidad Adventista de Bolivia UAB, 528, Cochabamba, Bolivia
apariducho@gmail.com
[2] Universidad Nacional de Educación UNAE, Chuquipata 030154, Ecuador
derling969@gmail.com, roxana.auccahuallpa@unae.edu.ec

Abstract. The objective of the study was to determine the level of understanding and characterize the type of ideal geometric thinking that teachers demonstrate in solving problems with the use of the GeoGebra software. The research was of mixed character, with sequential exploratory design. It was structured in two phases. The population was made up of 120 teachers who attended the course entitled: Innovate and transform the teaching of mathematics with GeoGebra. The virtual course was developed by the National University of Education during the period 2019–2020. In the first phase of the quantitative phase, a diagnostic questionnaire of descriptive analysis was applied. The second qualitative phase applied an analytical questionnaire on the polygons issued by the participants. The results show that the participants understood the activities in a dynamic way. To learn geometry, the level of understanding was very good when using the GeoGebra. It was possible to characterize that 15% of the participants developed a basic geometric representation without connection, demonstrating a submissive and linear style in the development of activities. 35% of the participants worked on contrasting geometric polygons, representing a constructivist design of visual variables. Finally, 50% of the participants designed axial geometric polygons, representing a higher level of complexity and totalitarian figuration. In conclusion, the software generated a great interest and motivation in the pedagogical activities. With the use of GeoGebra a level of effective understanding is acquired in the development of geometry.

Keywords: GeoGebra · Teaching and learning of mathematics · Technological innovation · Geometry · Continuous training

1 Introduction

Never before has information and communication technology (ICT) had such an impact on individual and collective development in the field of education. According to [1], before the Covid-19 pandemic, several countries were going through a period of digital transformation, but at different speeds and in different ways depending on social strata, geographical location or sectors of activity.

G. Rodriguez Morales et al. (Eds.): TICEC 2020, CCIS 1307, pp. 258–271, 2020.
https://doi.org/10.1007/978-3-030-62833-8_20

In the case of Ecuador, education was aware of this transformation. There was a change in communication habits, business, and access to services. From this perspective, the Ecuadorian Institute of GeoGebra, based at the National University of Education (UNAE), is developing training courses in the use of GeoGebra as a teaching resource for mathematics. Courses are directed to basic education within the Continuous Training Program for in-service teachers. The course is developed in a bimodal way: a classroom part and a virtual part.

With the pandemic caused by VOC-19 and confinement status, people's dependence on technology has increased [1]. The current situation requires teleworking activities and virtual classes. In view of this situation, based on the experience gained in the bimodal course, a continuing education course was offered entitled: "Innovate and transform the teaching of mathematics with GeoGebra" (ITEMG). The course [2] has 200 free places for in-service teachers. Finally, only 120 places were occupied.

The ITEMG course consists of nine units, covering different topics. Each unit is developed separately during fifteen days with a virtual introduction through the zoom, after applying autonomous and cooperative works among teachers and the feedback of the facilitators. ITEMG allows for consideration of the demands, challenges and difficulties faced by teachers in their pedagogical practice and teacher training [2].

This study is part of a research project entitled "The impact of the use of GeoGebra in the teaching and learning process of mathematics in Ecuadorian teachers" developed by the Institutional Research Group Eureka 4i with interdisciplinary, intercultural, international and innovative studies [3].

1.1 General Objective of the Research

Determine the level of understanding that students have with the use of the GeoGebra software. Then, characterize the type of geometric thinking that teachers demonstrate in solving problems with the use of GeoGebra.

1.2 Specific Objectives

1. Perform a theoretical, pedagogical and didactic analysis issued by different authors when using GeoGebra in the process of teaching and learning mathematics
2. Design and develop a descriptive diagnostic questionnaire
3. To determine the level of understanding that participants acquire in the ITEMG course
4. Characterize the type of geometric thinking emitted by ITEMG course participants in solving problems with the use of the GeoGebra software

2 Background

2.1 Information and Communication Technologies

In 1983, the Center for Educational Technology at Harvard University began using information and communication technologies (ICTs) in learning mathematics and science in primary and secondary education. In this context, the use of ICTs is widely accepted in different areas of today's society. Likewise, with respect to teaching and learning mathematical content, various researches indicate that the implementation of ICTs offers opportunities for students. Opportunities to develop activities that favour exploration, formulation of conjectures, verification, discovery of properties and generalisation of results. However, such technologies have not been installed in formal education as naturally as they are in other areas such as economics, medicine, and communication, among others [4].

According to [5, 6], learning mathematics is effective only when the student is able to identify conditions for the use of mathematical knowledge in new situations. Although mathematics has been taught and learned for millennia, the nature and quality of mathematical teaching and learning has not been seriously studied until the last century. Learning mathematics is considered to be related to problem solving, because the heart of mathematics is problem solving and is a means of identifying, exploring, testing, and communicating solution strategies. In other words, when a person solves problems, he or she enters into a process of exploring different representations, searching for patterns, invariants and relationships between mathematical objects, which allow the presentation of arguments, communication of results, formulation of questions and setting out of new problems.

On the other hand, in the last decades the integration of ICTs has opened new paths in the process of teaching and learning mathematics. These routes have aroused great interest from great mathematicians, researchers in mathematical education and mathematics teachers [7].

However, the integration of ICT alone is not enough, software already has an important place in innovative development, it can also have a significant potential for implementation in mathematics learning [8]. The implementation of mathematical software requires development, research and more detailed study. The appropriate and effective use of ICT can increase the level of visualization in mathematics and improve student learning, and its implementation can contribute to improving the quality of education [9].

Visualization of digital learning resources in lectures and practices allows students to better understand mathematical concepts and processes for meaningful learning [10]. Didactic resources strengthen communicative competence for both students and teachers. One of the ways to improve the visualization of abstractions in Mathematics is an effective and appropriate pedagogical and didactic application supported by ICTs [11].

2.2 GeoGebra

There are a variety of digital resources for teaching mathematics related to geometry, for example, the use of a Dynamic Geometry System (DMS), Geometr's Sketchpad and Cabri. But many of these software applications were not created with a particular pedagogical approach, nor are they all freely available or suggested in the curriculum for mathematics teaching.

GeoGebra software is a free open source program that combines features of dynamic geometry software, computer algebra systems and spreadsheet programs [12]. GeoGebra was created by Markus Hohenwarter as part of his Master's thesis in Mathematics

Education at the University of Salzburg (Austria) in 2002. With its unique ability to illuminate both algebraic and geometric concepts. GeoGebra provides a mechanism to improve how mathematics is taught and learned in classrooms at all levels of the education system [13].

According to [14], an important tool during exploration within GeoGebra is dragging. The basic principle of dragging an object from a figure is that the figure should keep all its properties as it was built. That is to say, it is the one that differs from a mathematical object constructed from a drawing. Because it is possible to observe how the attributes, length of a segment, area, amplitude of one or several angles change. of the mathematical objects involved. The importance of this dragging tool is that it allows observing invariants and patterns among the elements of a constructed figure [15].

2.3 Continuing Education of Teachers

According to [16] one of the problems in integrating ICTs and mathematics is teacher training. [17, 18] They say that teachers are considering using GeoGebra, emphasizing that the education system is changing from traditional to new, from analog to digital.

Similarly, [19] they say, there is a general tendency for teachers to evaluate themselves as not being trained to use ICTs and that they have them available in educational institutions. They are trained to handle them technically, although their level depends on the novelty of the technology. They claim that they handle them at home, but not in educational institutions.

In addition, according to [20], they indicate that they have little training in the design and production of media, other than in their didactic use. Regardless of variables such as age and gender, teachers generally show great interest in being trained to use these didactic instruments. Although it is logical, as it happens with other variables, younger teachers are more concerned about their incorporation, use and training than older ones. They admit that they have not received a real qualification throughout their studies, to incorporate into their professional practice.

Research on the use of ICTs in teacher training is abundant. However, very little addresses the use of GeoGebra in teacher training, especially as it relates to Ecuadorian teachers. Therefore, the research emphasizes the use of GeoGebra software in the teaching and learning process of geometry in Ecuadorian teachers.

2.4 Ecuadorian Institute of GeoGebra

The Ecuadorian Institute of GeoGebra (IEG) is based at the National University of Education (UNAE). It was officially constituted on April 24, 2018, on the occasion of the celebration of the 6th Ibero-American GeoGebra Day. The objective of creating the Ecuadorian Institute of GeoGebra is to contribute to the purposes and functions of the International Institute of GeoGebra. The functions of the institute are to motivate, promote, support, and disseminate the use of GeoGebra as a powerful tool to innovate and transform the teaching of mathematics at all educational levels in Ecuador. GeoGebra is more than just dynamic mathematics software; it is an educational program that links the Science Technology, Engineering and Mathematics STEM project.

3 Research Method

3.1 Research Focus

The methodology of the research was of a mixed or multi-method type. The research design was "Field study". A theoretical review and documentary analysis was applied. These reviews allowed to complement, define and describe the object of study. It was possible to analyse and distribute the research time, without having to influence the variables involved. In this way, the use of the quantitative and qualitative paradigm is implied, a method recommended by [21].

3.2 Research Design

There are many multi-method research designs. The authors applied the Deplix method, defined by [22] as a sequential exploratory design. This design is done in two phases. In the first phase of a quantitative nature, numerical data are collected. Then the results are analyzed in an explanatory way. Based on the numerical data, the second phase begins. In the second phase, complementary qualitative data is collected and interpreted to meet the objectives set.

3.3 Population and Sample

The study population corresponds to the participants of the continuing education course "ITEMG" for the four-month period May-August 2020. Directly, the sample was 120 teachers (62 females and 48 male). All the participants in the course represented 17 provinces of Ecuador. The provinces of Azuay, Bolivar, Cañar, Carchi, Chimborazo, El Oro, Esmeraldas, Guayas, Loja, Los Ríos, Manabí, Napo, Orellana, Pastaza, Pichincha, Santo Domingo and Tungurahua. A non-probabilistic, purposive sample was established for the sample, no statistical formula was applied and all participants were chosen for the study [23].

3.4 Research Instruments

In order to respond to specific objective 2, a descriptive diagnostic questionnaire was designed. The questionnaire allowed the collection of data in the quantitative phase. The questionnaire measures the ease and understanding that GeoGebra provides in mathematics education. The questions were sent by e-mail to all the participants of the study. The answer options were of a scalar type. The very poor option has the lowest value with 1 PV point. Then the poor option has a value of 2 "D" points. The regular option was the intermediate level with 3 "R" points. The very good option had a score of 4 "VG". Finally, the excellent option with the score greater than 5 "E" points (see Table 1). A pilot test was applied to 5 participants to evaluate the level of reliability of the instrument. Cronbach's alpha coefficient was applied to the instrument for reliability analysis. The statistical value was 0.838 as [24] it is demonstrated that it is a reliable instrument.

Table 1. Table 1. Diagnostic questionnaire applied to ITEMG course participants.

Universidad Nacional de Educacion
Docente del Grupo de Investigación Institucional EUREKA 4i

UNAE

Dear Teacher, This questionnaire aims to determine teachers' perceptions about learning mathematics with the use of the GeoGebra® software from the development of the course "Innovate and transform math teaching with GeoGebra®". The information you provide is an important contribution to further improve the development of the course. We kindly ask for your participation and guarantee the anonymity of your answers.

Items	On a scale of 1 to 5, how do you rate the premise 'The use of GeoGebra software favours:'?	Options				
		1 VP	2 D	3 R	4 VG	5 E
1	Understanding math concepts					
2	The development of critical and analytical thinking					
3	Mathematical logical reasoning					
4	Numerical reasoning					
5	Mathematical demonstrations in a dynamic way					
6	The verification of conjectures and mathematical postulates					
7	The discovery of objects and mathematical concepts					
8	Meaningful learning of mathematics					
Items	**On a scale of 1 to 5, how do you rate the premise 'When using the GeoGebra® in the classroom the software allows**	**Options**				
		1 VP	**2 D**	**3 R**	**4 VG**	**5 E**
9	To awaken students' interest and motivation in mathematics					
10	The development of skills for teamwork/collaborative work					
11	Understanding math concepts					
12	Develop positive and supportive attitudes towards learning mathematics					
13	The discovery of objects and mathematical concepts in the classroom.					
14	Understanding mathematics in a dynamic way					
15	The verification of postulates and concepts of abstract mathematics.					
16	A creative and dynamic construction of mathematical concepts					

The data obtained from the questionnaire were analysed in a descriptive manner. Based on the results, a second instrument of qualitative analysis is designated. The participants were asked to create from any polygon, to build a new polygon on one side less and whose area is equal to the initial polygon. In this way, it was possible to analyze and characterize the type of geometrical thinking emitted by the participants of the ITEMG course (see Table 2).

Table 2. Qualitative analysis questionnaire applied to ITEMG course participants

<table>
<tr><td colspan="2">Universidad Nacional de Educacion
Docente del Grupo de Investigación Institucional EUREKA 4i</td><td>UNAE</td></tr>
<tr><td colspan="3">Dear teacher, the purpose of this application is to identify the most common geometric models suitable for use with GeoGebra® software, based on the development of the course "Innovating and transforming mathematics teaching with GeoGebra®". The information you provide is an important contribution to further improve the development of the course. We kindly ask for your participation and guarantee the anonymity of your answers.</td></tr>
<tr><td>Geometrical development</td><td colspan="2">Figure</td></tr>
<tr><td>From any polygon, build a new polygon on one side less and whose area is equal to the initial polygon.</td><td colspan="2"></td></tr>
</table>

The analysis of the activity proposed in Table 2 was developed in an interpretative way. The researchers developed the triangulation of analytical and theoretical data from different authors on the figures designed by the participants. The results are presented below.

4 Analysis of the Results

4.1 Diagnostic Phase - Quantitative

After developing the ITEMG course, the researchers applied the Diagnostic Questionnaire (see Table 1). The results of the premise "The use of the GeoGebra software favors" are shown in Fig. 1. The results of each item were averaged through an arithmetic mean, the highest average being the "very good" option with an average of 42.08%. Second, the regular option with an average of 35.72%. The excellent option averaged 16.14%. Then the poor option with an average of 5.41% and the lowest was 0.62% for the very poor option.

Finally, it can be indicated that the majority of participants consider that the use of GeoGebra greatly favors demonstrating and applying mathematical concepts. Similarly for [25], the development and didactic use of GeoGebra facilitates the generation of user-specific concepts. The manipulation of its graphic representation; later, through the identification of regularities observed in the graph, the algebraic expression is constructed as a way to represent the relationships between the quantities that intervene in the quantities of the situation.

This type of sequence is important because mathematical concepts such as the function are introduced with the graphical representation, based on experimentation with the software. GeoGebra offers algebraic representations that facilitate the development of critical and analytical thinking. According to [26], the GeoGebra system is a phenomenon that allows mathematical demonstrations to be made dynamically. Tools such as the GeoGebra software are useful resources in the mathematics classroom because

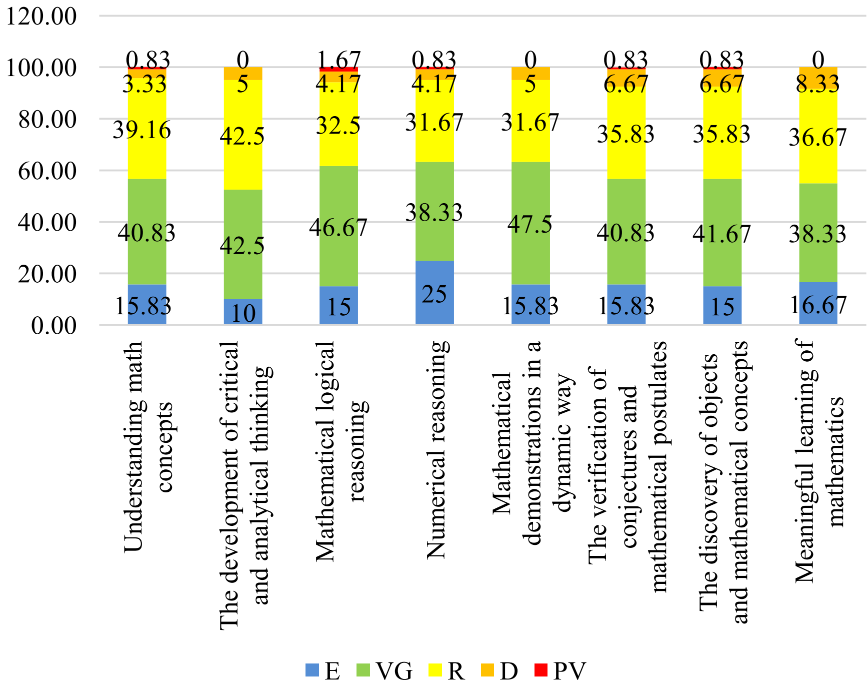

Fig. 1. Results of the premise'The use of GeoGebra software favours' applied to research participants.

they allow students to show, by putting into practice their previous knowledge, what they have been able to internalize so far.

The second premise, "Using the GeoGebra® software in the classroom allows", focused on the understanding and motivation that the software generates in the students. Similarly, when applying an average of the items, the first result was 46.77% considered very good. In second place the excellent option with an average of 26.87%. Then, the regular option with an average of 21.45%. The deficient option with an average value of 4.47%, the option with the lowest average was "very deficient" with a value of 0.41%. In this way, it can be described that the use of geography awakens interest and motivation in the student body. Like [14], it is established that geography is a dynamic component of exposure to and understanding of mathematical content. Creative construction depends on the teacher and his or her development, that is, the teacher must possess the basic skills of use in order to develop dynamic activities (see Fig. 2).

The dynamic activities in the mathematics classroom facilitate understanding [27]. For the researchers of the ITEMG course the results show that motivation improves academic performance. Students' performance can be positively affected by improving the development of their mathematical thinking and the level of competence in responding to problems associated with calculus and algebra. Finally, the third specific objective is answered, determining that the level of understanding acquired by ITEMG course

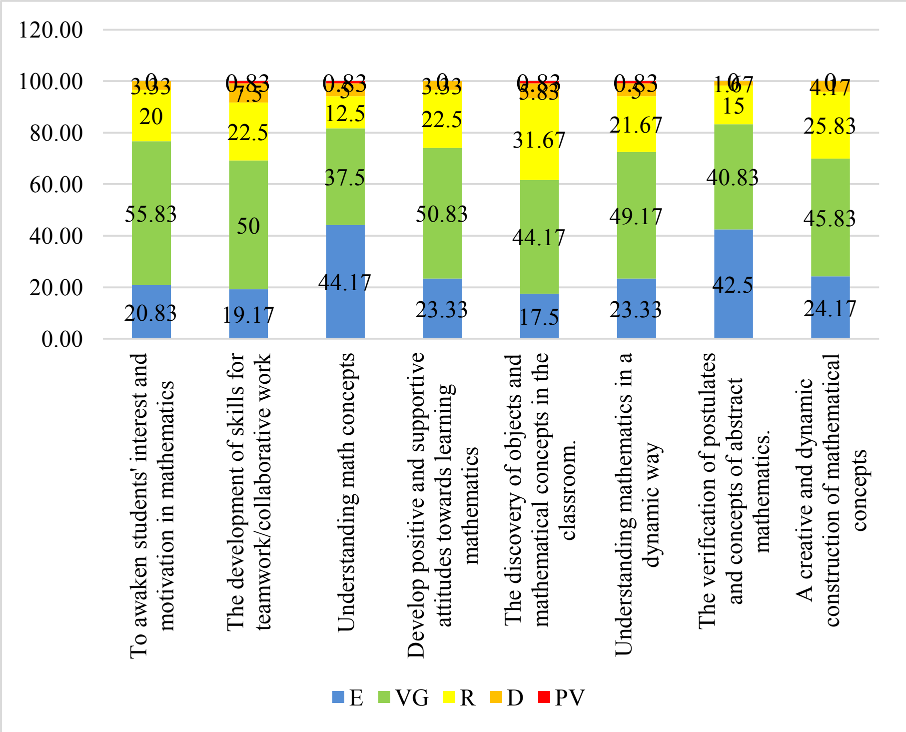

Fig. 2. Results from the premise 'When using GeoGebra® in the classroom the software allows' applied to research participants.

participants is very good. The "very good" level covers the competencies of developing critical thinking, logical reasoning, meaningful learning, dynamic creative teaching, motivation and understanding in the process of teaching and learning through the use of the GeoGebra software.

4.2 Qualitative - Interpretative Phase

After determining the level of understanding of the participants, the qualitative analysis questionnaire was executed to answer specific objective 4. In Table 2, it can be seen that the proposed activity was to design from any polygon, a new polygon on one side less and whose area is equal to that of the initial polygon. The analysis of the development of the activity as the geometric representation was presented with the following results:

Basic Geometric Representation without Connection

15% of the participants drew two polygons, so the second polygon has a smaller side than the first and its areas are equal. But by moving any of the free points of the polygons, the equality of area is no longer achieved. As shown below (See Fig. 3) (Table 3).

Graphic representation is understood more in line with the current concept of "meaning", which includes the emotional, social and cultural dimensions of the designer [28].

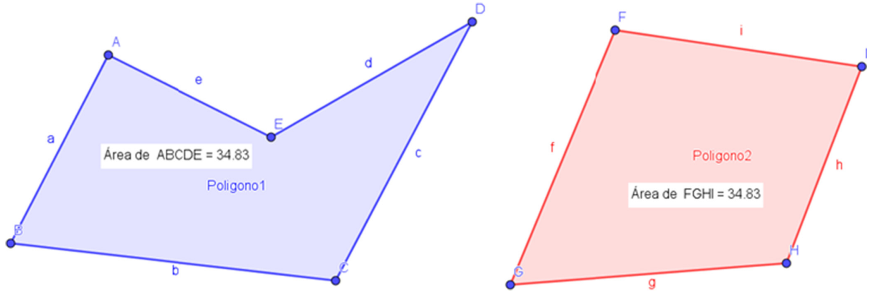

Fig. 3. Polygons drawn by participants with distant and unconnected geometric figures.

Table 3. Number of attempts to construct polygons with equal area

Number of attempts	Difficulties	Percentage
Those who managed on the first attempt	They had no difficulty	45%
Those who managed on the second attempt	Most have drawn and not built or were not related to each other	30%
Those who managed on the third attempt	For them, a second feedback was necessary	20%
Those who didn't make it	Despite the suggestion, they did not manage to carry out the activity as requested	5%
TOTAL		100%

In a certain sense, both terms could even be considered synonyms, although on this occasion the authors apply the representation of polygons in an interpretative manner with respect to their model of geometric thought in order to avoid confusion.

The two polygons in Fig. 3 represent a basic differentiation and organization from the topological point of view. The two polygons are distant and are characterized in the unconscious plane of a simple model. The teacher represents aspects of behaviorism and linear thinking, i.e., he only designs the requested figures without adding links between the polygons.

Contrast Geometric Representation. Another characteristic of the teachers in the development of the proposed activity was the use of connection or contrast by means of the vertices. It has been observed that 35% of them use the more classic or common figures such as the square and the triangle, that is, they lack that generalized understanding of any polygon. As shown in Figs. 4 and 5.

Contrast geometry is defined by the interposition of two or more figures, also when placing an image in front of or behind another one [29]. In Fig. 4, the participants placed contrasting polygons, between a square and a triangle, the figures are unified in common side GB and vertex B. Similarly, in Fig. 5, participants contrasted a triangle and

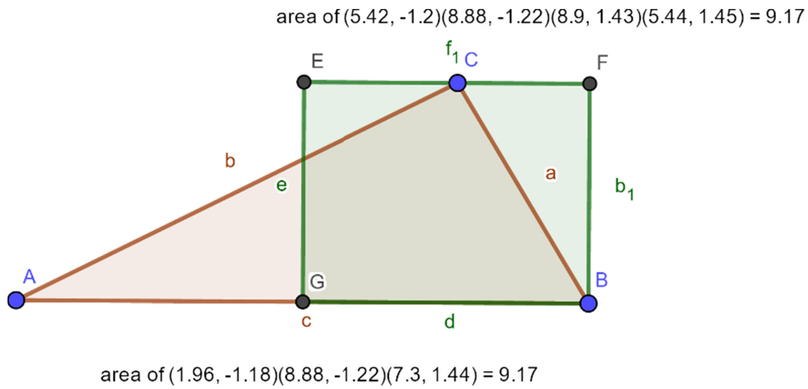

Fig. 4. Equal area of square and triangle developed by IEMTEG participants

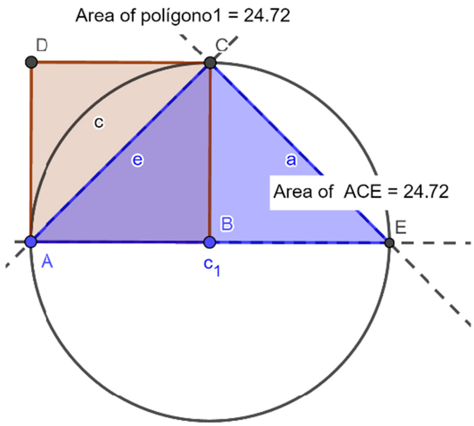

Fig. 5. Equal area of the quadrilateral (ABCD) and triangle (ACE) contrasted in a circle

a square, both circled by a circle. As for the perception of objects and levels of perception, the author demonstrates the qualities of geometric abstraction even if modifications are made. The author generates visual variables by establishing new parameters in his design. In the same way, at a constructivist level the author complies with the requirement of equality of areas, but his sides and vertices are interconnected in a dependent way. These two examples show the creativity generated by the author when applying GeoGebra. The perceptive constancy of form and value can also be appreciated by looking for models that are different from the common patterns of separate figures.

Axial Geometric Representation. Finally, 50% of the figures were of axial geometric design (see Fig. 6).

The level of axial geometric representation is constituted by the connection between the brain and the world, whether with respect to nature, others and oneself [30]. Figure 6 leads to an objective representation of the geometric continuity of polygons. It is perceived that the author has the capacity to justify two or more figures around two axial

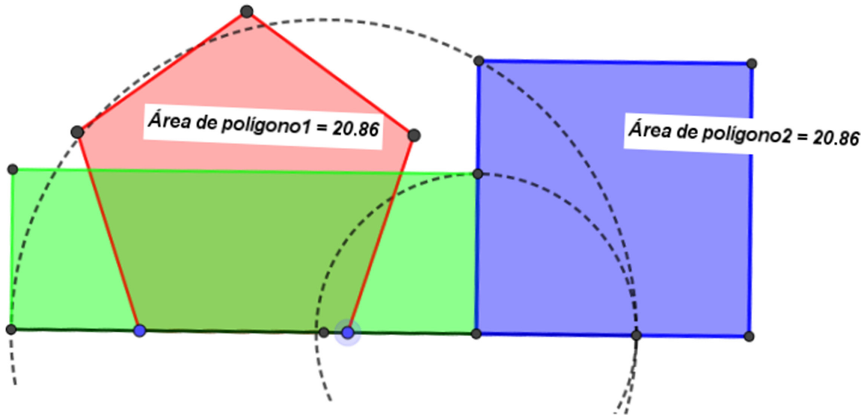

Fig. 6. Axial geometric representation in the equal area of three polygons (Color figure online)

axes. The polygons emitted by the activity were the pentagon and the square with equal area, but, to establish their design a green rectangle was added. All subscribed to a common axis.

Within the thematic part it is necessary to structure that the polygons are visually connected and they can be moved without difficulty that there is a division between them. These geometric designs represent a hierarchy of complexity and depth. At a cognitive level, the order and dynamic sequence of the course object is perceived. An order is also established to emphasize one or several structures that link polygons and other figures that are attached.

5 Conclusions

In conclusion, GeoGebra is one of the most important software since it facilitates and helps teachers to interact dynamically with thematic contents in the area of mathematics; this program is one of the technological options that enriches the quality of research and visualizes mathematics from different perspectives, supporting feedback; it also offers teachers strategies for instruction according to the needs of students. It also facilitates learning through virtual representations that are representations of reality and concentrates pedagogical benefits.

After analyzing the results in the quantitative phase, it is possible to describe that the participants acquire a level of effective understanding in the development of geometry. It has been proved that learning environments are not only physical spaces, tools or time distributions, but the result of the teacher-learner relationship from a digital didactic environment, versatile and according to the participants' degree of maturity, rich in significant activities with permanent changes, multiple resources and completely dynamic. GeoGebra also provided the motivation in students during the course, establishing a digital connectivity between teachers and students.

An effective and sustainable education is an innovative education that uses ICT and other means to improve the teaching and learning process in general and in mathematics in particular. At the same time, it seeks to meet the needs of current and future generations in relation to motivational, social and environmental aspects. In this way, teachers are motivated in their learning and continuous training not only in the use of GeoGebra, but in different areas or dimensions such as disciplinary, pedagogical, technological and human relations for the sustainable use of ICTs to raise the quality of education and life in the long term.

With respect to the characterization of the type of geometric thinking, three groups were structured. 15% designed basic unconnected geometric representations, demonstrating a submissive and linear style in the development of activities. 35% of the participants worked on contrasting geometric polygons, representing a constructivist design of visual variables. Finally, 50% of the participants designed axial geometric polygons, representing a higher level of complexity and totalitarian figuration.

References

1. García, F.: El sistema universitario ante la COVID-19: Corto, medio y largo plazo. En Universidad (2020). https://bit.ly/2YPUeXU
2. Universidad Nacional Abierta: Curso virtual, "Innovar y transformar la enseñanza de las matemáticas con GeoGebra (2020). https://oeiecuador.org/uploads/files/news/OEI-Education/135/curso-de-GeoGebra-formacion-continua-unae.pdf
3. Universidad Nacional de Educación UNAE: Eureka 4i (2016). https://unae.edu.ec/oferta/eureka-4i/
4. Artigue, M.: Tecnología y enseñanza de las matemáticas: desarrollo y aportaciones de la aproximación instrumental. Cuadernos de Investigación y Formación en Educación Matemática **6**(8), 13–33 (2011)
5. Kilpatrick, J.: History of research in mathematics education. Encyclopedia of Mathematics Education, Springer, Cham (2020)
6. Schoenfeld, A.: Mathematics Problem Solving. Academic Press, New York (1985)
7. Pari, A.: El impacto de GeoGebra en el Desarrollo profesional del profesorado de matemáticas. Publisher UNAE, Ecuador (2019)
8. Mendoza, D.J., Mendoza, D.I.: Information and communication technologies as a didactic tool for the construction of meaningful learning in the area of mathematics. Int. Electro. J. Math. Edu. **13**(3), 261–271 (2018). https://doi.org/10.12973/iejme/3907
9. Mendoza, D., Nieto, Z., Vergel, M.: Technology and mathematics as a cognitive component. J. Phys: Conf. Ser. **1414**, 012007 (2019). https://doi.org/10.1088/1742-6596/1414/1/012007
10. Ballester, A.: Meaningful Learning in Practice. J. Educ. Hum. Dev. **3**(4), 199–209 (2014)
11. Fernández, R., Delgado, L.: La realidad aumentada como recurso didáctico para la enseñanza de las matemáticas. Ciencias Aplicadas **10**(7), 2560 (2020)
12. Sarmiento, W., Luna, K.: GeoGebra application of software in mathematics practices under a constructivist methodology. Revista Killkana Sociales **1**(2), 45–50 (2017)
13. Zengin, Y.: Incorporating the dynamic mathematics software GeoGebra into a history of mathematics course. Int. J. Math. Educ. Sci. Technol. **49**(7), 1083–1098 (2018). https://doi.org/10.1080/0020739X.2018.1431850
14. Iiris, A., Kjell, B., Mirko, R.: Generating the patterns of variation with GeoGebra: the case of polynomial approximations. Int. J. Math. Educ. Sci. Technol. **47**(1), 45–57 (2016)
15. Umit, K.: Turkish mathematics teachers' experiences with GeoGebra activities: changes in beliefs. Res. Math. Educ. **14**(3), 293–294 (2012)

16. Llorente, M.: Aspectos Fundamentales de la Formación del Profesorado en TIC. En Pixel-Bit. Revista de Medios y Educación **31**, pp. 121–130 (2008) https://idus.us.es/bitstream/handle/11441/14245/file_1.pdf?sequence=1
17. García, A., Tejedor, J.: Condiciones (actitudes, conocimientos, usos, intereses, necesidades formativas) a tener en cuenta en la formación del profesorado en TIC. Enseñanza **23**, 115–142 (2005)
18. Monedero, J.: Uso y evaluación de materiales educativos durante el desarrollo del curriculum: ¿Qué hacen los profesores?, ¿Qué pueden hacer? Pixel-Bit **12**, 55–64 (1999)
19. Carrillo, A., Llamas, I.: GeoGebra. Mucho más que geometría dinámica. Alfaomega Ra-Ma, México (2009)
20. Fernández, M., Cebreiro, B.: La integración de los medios y nuevas tecnologías en los centros y practicas docentes. Pixel-Bit **20**, 33–42 (2003)
21. Yanez, S., Martinez, M., Mendoza, D., Lucano, S., Sanchez, H., Barros, R.: The solitary and hatred within social media. Acad. J. Interdisc. Stud. **8**(3), 71–80 (2019)
22. Hernández, R., Fernández, C., Baptista, P.: Metodología de la investigación. McGraw Hill, México (2010)
23. Wright, S., Noble, R., Bailer, A.: Equal-precision allocations and other constraints in stratified random sampling. J. Stat. Comput. Simul. **77**(12), 1081–1089 (2007). https://doi.org/10.1080/10629360600897191
24. Pranav, D., Naved, A., Divya, G.: Genetically modified organisms: Reliability analysis and perceptions. J. Inf. Optim. Sci. **39**(7), 1401–1415 (2018)
25. Royati, A., Ahmad, F., Rohani, A.: The effects of GeoGebra on mathematics achievement: enlightening coordinate geometry learning. Procedia Soc. Behav. Sci. **8**, 686–693 (2010)
26. Martin, D.: Student's Guide to the Study, Practice, and Tools of Modern Mathematics. Chapman and Hall/CRC, New York (2010)
27. Gallego, F., Granados, H., Sanchez, O.: Influence of GeoGebra in the motivation and self-regulation of the learning of calculus and algebra in universities. Espacios **39**(17), 7–20 (2018)
28. Asghar, J., Mohammad, K., Mansour, K.: Representational approaches in graphic design. Bagh- e Nazar **14**(50), 49–56 (2017)
29. Monedero, J.: Simulación visual de materiales: teoría, técnicas, análisis de casos. Publicacions Academiques Digitals de la UPC, Barcelona (2015)
30. Castellaro, M.: El concepto de representación mental como fundamento epistemológico de la psicología. Límite **6**(24), 55–67 (2011)